Lambda-F: A Two-Signal Framework for Market Regime Detection*

Combining Commutator-Based Rotation Detection with Correlation Synchronization
and Game-Theoretic Enhancement

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Abstract

We present Lambda-F (Λ_F), a novel framework for detecting market regime shifts through the analysis of factor covariance matrix dynamics using concepts from differential geometry. The core innovation lies in computing the commutator between the instantaneous factor covariance tensor $\mathbf{F}_t$ and its temporal derivative $\dot{\mathbf{F}}_t$, capturing the non-commutativity that emerges when market factors undergo coordinated rotations—a signature of institutional repositioning preceding major market transitions.

A key methodological contribution is the **two-signal detection system**: Lambda-F detects factor rotation (institutional repositioning), while a complementary correlation signal detects synchronized selloffs (panic). This combination achieves comprehensive coverage of institutional events. We further introduce **ex-ante percentile computation** using trailing windows, ensuring all results are achievable in real-time deployment without information leakage.

The framework is extended with a game-theoretic layer incorporating Von Neumann-Nash equilibrium concepts and Soros reflexivity theory, creating a two-dimensional classification system that distinguishes between regime shifts with high crash probability (Q4) and regime shifts with lower immediate risk (Q3).

Empirical validation across **33 major market events spanning 2000-2024** in 8 asset classes achieves **100% detection rate** with the two-signal system, while correctly excluding 4 exogenous black swan events (5 for Emerging Markets, which correctly detects COVID as rotation). Marginal exceedance rates match nominal percentiles by construction; the non-trivial finding is that elevated signals cluster around genuine regime transitions rather than appearing randomly. The framework provides actionable signals with lead times of 14-90 days.

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Keywords: Market regime detection, factor rotation, commutator functional, correlation synchronization, two-signal detection, game theory, reflexivity, ex-ante percentiles

Contents

1 Introduction

The prediction of market regime transitions represents one of the most challenging problems in quantitative finance. Traditional approaches rely on volatility clustering models (GARCH family), regime-switching specifications (Hamilton, 1989), or technical indicators that often generate excessive false positives. We propose a fundamentally different approach grounded in the mathematical physics of dynamical systems, specifically the detection of non-commutative dynamics in factor covariance evolution.

1.1 Motivation and Problem Statement

Financial markets exhibit complex dynamics driven by the interaction of multiple agents with heterogeneous strategies, time horizons, and information sets. During periods of market stress, these interactions can produce coordinated repositioning across institutional portfolios, manifesting as rotations in the factor covariance structure. The central hypothesis of this work is:

> *Major market regime transitions are preceded by detectable non-commutativity in the evolution of factor covariance matrices, reflecting coordinated institutional strategy rotation that breaks the normal quasi-equilibrium dynamics.*

This hypothesis draws from two distinct intellectual traditions:

1. **Differential Geometry:** The commutator $[\mathbf{F}, \dot{\mathbf{F}}]$ measures the failure of infinitesimal operations to commute, a fundamental concept in Lie algebra that characterizes rotational dynamics in physical systems.
2. **Game Theory:** Market crashes occur when Nash equilibria become unstable and agents' strategies exhibit reflexive feedback loops (Soros, 1987), creating cascade dynamics.

By synthesizing these frameworks, we construct a detection system that captures both the geometric signature of coordinated factor rotation and the game-theoretic conditions necessary for that rotation to precipitate a crash.

1.2 Key Innovation: Two-Signal Detection

Initial validation of the Lambda-F framework revealed an important gap: while highly effective at detecting factor rotations, Lambda-F missed certain market events characterized by synchronized selloffs where all factors moved together without rotation. This led to our key innovation:

- **Lambda-F (Rotation):** Detects institutional repositioning where capital flows *between* factors
- **Correlation (Synchronization):** Detects panic selloffs where all factors decline *together*

The combination achieves 100% detection across 33 major market events spanning 2000-2024, while correctly excluding exogenous black swan events.

1.3 Contributions

This paper makes the following contributions:

1. **Lambda-F Framework:** We introduce a commutator-based functional $\Lambda_F(t)$ that quantifies the non-commutativity of factor covariance evolution, providing a mathematically principled measure of regime instability.
2. **Two-Signal Detection System:** We combine Lambda-F with a correlation synchronization signal, achieving comprehensive coverage of institutional market events.
3. **Ex-Ante Percentile Computation:** We introduce trailing-window percentile rankings that ensure real-time deployability without information leakage.
4. **Game-Theoretic Enhancement:** We extend the pure geometric framework with reflexivity indicators (funding rates, sentiment indices) to distinguish high-probability crash scenarios from benign regime rotations.
5. **Comprehensive Validation:** We demonstrate 100% detection rate across 33 events in 8 markets spanning 25 years (2000-2024), with black swans correctly excluded.
6. **Practical Implementation:** We provide complete algorithmic specifications enabling practitioners to implement the framework for real-time monitoring.

1.4 Paper Organization

Section 2 develops the theoretical foundations. Section 3 presents the two-signal detection system. Section 4 describes the computational methodology. Section 5 provides comprehensive empirical validation. Section 6 introduces the game-theoretic enhancement layer. Section 7 discusses limitations and future directions. Section 8 concludes.

2 Theoretical Framework

2.1 Factor Covariance Dynamics

Let $\mathbf{r}_t \in \mathbb{R}^n$ denote the vector of asset returns at time t. In the factor model framework, returns are generated by exposure to k systematic factors:

$$\mathbf{r}_t = \mathbf{B}\mathbf{f}_t + \boldsymbol{\epsilon}_t \tag{1}$$

where $\mathbf{B} \in \mathbb{R}^{n\times k}$ is the factor loading matrix, $\mathbf{f}_t \in \mathbb{R}^k$ is the vector of factor returns, and $\boldsymbol{\epsilon}_t$ represents idiosyncratic noise.

The factor covariance matrix is defined as:

$$\mathbf{F}_t = \mathrm{Cov}(\mathbf{f}_t|\mathcal{I}_{t-1}) \tag{2}$$

where $\mathcal{I}_{t-1}$ denotes the information filtration (sigma-algebra) up to time $t-1$. Note: we use $\mathcal{I}$ for the filtration to avoid confusion with the factor covariance matrix $\mathbf{F}$.

In practice, we estimate $\mathbf{F}_t$ using a rolling window of factor returns with exponential weighting:

$$\hat{\mathbf{F}}_t = \sum_{\tau=0}^{W-1} \omega_\tau (\mathbf{f}_{t-\tau} - \bar{\mathbf{f}}_t)(\mathbf{f}_{t-\tau} - \bar{\mathbf{f}}_t)^\top \tag{3}$$

where $\omega_\tau \propto \exp(-\tau/\lambda_{EMA})$ are exponentially decaying weights and W is the window size.

2.2 The Commutator Functional

The central innovation of our framework is the commutator between the factor covariance tensor and its temporal derivative.

Definition 2.1 (Lambda-F Functional)**.** *The Lambda-F functional at time t is defined as:*

$$\Lambda_F(t) = \frac{\left\| [\mathbf{F}_t, \dot{\mathbf{F}}_t] \right\|_F}{\|\mathbf{F}_t\|_F \cdot \left\| \dot{\mathbf{F}}_t \right\|_F + \epsilon} \tag{4}$$

where $[\cdot,\cdot]$ denotes the matrix commutator:

$$[\mathbf{F}_t, \dot{\mathbf{F}}_t] = \mathbf{F}_t \dot{\mathbf{F}}_t - \dot{\mathbf{F}}_t \mathbf{F}_t \tag{5}$$

$\|\cdot\|_F$ is the Frobenius norm, and $\epsilon > 0$ is a small regularization constant.

The temporal derivative is approximated by finite differences:

$$\dot{\mathbf{F}}_t \approx \mathbf{F}_t - \mathbf{F}_{t-1} \tag{6}$$

2.3 Geometric Interpretation

The commutator $[\mathbf{F}, \dot{\mathbf{F}}]$ has profound geometric significance. When $\mathbf{F}$ and $\dot{\mathbf{F}}$ commute (i.e., $[\mathbf{F}, \dot{\mathbf{F}}] = 0$), the eigenvectors of the covariance matrix remain fixed while only eigenvalues change. This corresponds to volatility scaling without structural change.

When $[\mathbf{F}, \dot{\mathbf{F}}] \neq 0$, the eigenvectors (factor directions) are rotating in the space of covariance matrices. This rotation indicates that the relationships between factors are changing—a signature of regime transition.

Theorem 2.2 (Eigenbasis Identity)**.** *Let $\mathbf{F}_t = \mathbf{V}_t \mathbf{D}_t \mathbf{V}_t^\top$ be the eigendecomposition of $\mathbf{F}_t$ with eigenvalues $d_1, \ldots, d_k$. Working in the instantaneous eigenbasis of $\mathbf{F}_t$:*

$$([\mathbf{F}_t, \dot{\mathbf{F}}_t])_{ij} = (d_i - d_j)(\dot{\mathbf{F}}_t)_{ij} \tag{7}$$

and therefore:

$$\left\| [\mathbf{F}_t, \dot{\mathbf{F}}_t] \right\|_F^2 = 2 \sum_{i<j} (d_i - d_j)^2 \left| (\dot{\mathbf{F}}_t)_{ij} \right|^2 \tag{8}$$

since the commutator is skew-symmetric when both $\mathbf{F}_t$ and $\dot{\mathbf{F}}_t$ are symmetric.

This identity shows that Λ_F is a *gap-weighted measure of off-diagonal energy* of $\dot{\mathbf{F}}_t$ in the eigenframe of $\mathbf{F}_t$. The signal is large when:

1. Eigenvalue gaps are large (distinct factor importance)

2. Off-diagonal mixing is large (factor structure changing)

Crucially, this formulation avoids explicit reference to eigenvector derivatives $\dot{\mathbf{V}}_t$, eliminating gauge-dependence issues and numerical instability near eigenvalue degeneracies.

2.4 Connection to Lie Theory

The space of symmetric positive definite matrices $\mathcal{S}_k^+$ forms a Riemannian manifold. The commutator structure connects to the Lie algebra $\mathfrak{gl}(k)$ through the tangent space at $\mathbf{F}_t$.

Proposition 2.3 (Non-Commutativity Measure). *The quantity* $[\mathbf{F}_t, \dot{\mathbf{F}}_t]$ *is a non-commutativity measure of the covariance evolution: it vanishes if and only if the evolution is diagonal in a fixed eigenframe, i.e., when* $\mathbf{F}_t$ *and* $\dot{\mathbf{F}}_t$ *can be simultaneously diagonalized.*

Geometrically, $[\mathbf{F}, \dot{\mathbf{F}}]$ quantifies the *failure of simultaneous diagonalizability* along the path $\gamma : [0, T] \to \mathcal{S}_k^+$—equivalently, the rate of eigenframe rotation.

3 Two-Signal Detection System

Initial validation revealed that Lambda-F, while highly effective at detecting factor rotations, missed certain market events where all factors moved together without rotation. Analysis of these failures motivated a two-signal approach.

3.1 Signal Complementarity

Definition 3.1 (Two-Signal System). *The regime detection system comprises two complementary signals:*

1. ***Lambda-F (Rotation):*** *Detects coordinated institutional repositioning where capital flows between factors*

$$\Lambda_F(t) = \frac{\|[\mathbf{F}_t, \dot{\mathbf{F}}_t]\|_F}{\|\mathbf{F}_t\|_F \|\dot{\mathbf{F}}_t\|_F + \epsilon} \tag{9}$$

2. ***Correlation (Synchronization):*** *Detects panic selloffs where all factors decline together*

$$\rho_t = \frac{2}{k(k-1)} \sum_{i<j} corr_W(f_i, f_j) \tag{10}$$

where k is the number of factors and W is a trailing window (optimal: 21 days).

3.2 Intuition

Consider the following analogy:

- **Volatility** tells you how fast the car is going
- **Lambda-F** tells you the steering wheel is jerking (rotation)
- **Correlation** tells you all cars on the highway are swerving the same direction (synchronized panic)

3.3 Detection Logic

Definition 3.2 (Combined Regime Classification).

$$\begin{aligned} \textit{CRITICAL:} \quad & P_t^{\Lambda} \geq 90 \ \textit{OR} \ P_t^{\rho} \geq 95 && (11) \\ \textit{ELEVATED:} \quad & P_t^{\Lambda} \geq 75 \ \textit{OR} \ P_t^{\rho} \geq 90 && (12) \\ \textit{Normal:} \quad & \textit{otherwise} && (13) \end{aligned}$$

where P_t^{Λ} *and* P_t^{ρ} *are ex-ante percentile rankings.*

The signal source is indicated: (L) = Lambda-F, (C) = Correlation, (LC) = Both.

3.4 Events Requiring Each Signal

Table 1: Signal Type by Event Category

Event Type	Primary Signal	Examples
Gradual institutional rotation	Lambda-F	GFC 2008, 2022 Bear, 2014 Oil Bust
Synchronized panic/selloff	Correlation	Q4 2018 US, UK Mini-budget, Germany
Maximum stress (both)	Lambda-F + Correlation	2011 Eurozone Crisis

3.5 Ex-Ante Percentile Computation

A critical methodological requirement is ensuring that all percentile computations use only information available at the time of computation, preventing look-ahead bias.

Definition 3.3 (Ex-Ante Percentile — No Look-Ahead Guarantee).

Remark 3.4 (Critical Implementation Detail). *No future information leakage*

All published detection results are achievable in real-time deployment

Percentile calibration adapts to evolving market dynamics

Important: Event labels (cycle top, regime shift) are defined using forward-looking criteria for evaluation purposes, but the *signal itself* is computed causally using only past data available at each time t.

4 Methodology

4.1 Factor Construction

We distinguish two types of factor bases used in this framework:

Definition 4.1 (Factor Types).

- ***Constructed Factors:*** *Long-short style factors built from asset cross-sections (e.g., momentum, size, volume factors). These capture systematic return drivers through portfolio construction.*

- ***Proxy Factors:*** *Returns of liquid tradable instruments that serve as basis vectors for a market (e.g., sector ETFs for equities, commodity ETFs for commodities). These provide direct market exposure without portfolio construction.*

The factor covariance matrix $\mathbf{F}_t$ *is computed from whichever factor basis is appropriate for the market structure.*

For cryptocurrency markets, we use *constructed factors* from the cross-section of major assets (BTC, ETH, LTC, XRP):

1. **Market Factor (CMKT):** Equal-weighted average return across all assets:

$$\mathrm{CMKT}_t = \frac{1}{n} \sum_{i=1}^{n} r_{i,t} \tag{15}$$

2. **Cross-Sectional Spread (CSPREAD):** Performance-based long-short factor:

$$\mathrm{CSPREAD}_t = \bar{r}_t^{\mathrm{top}} - \bar{r}_t^{\mathrm{bottom}} \tag{16}$$

   where assets are ranked by trailing 30-day performance. Note: this is *not* a size factor in the Fama-French sense, but rather a spread between recent winners and losers within the crypto cross-section.

3. **Momentum Factor (CMOM):** 14-day momentum long-short:

$$\mathrm{CMOM}_t = \bar{r}_t^{\mathrm{winners}} - \bar{r}_t^{\mathrm{losers}} \tag{17}$$

4. **Volume Factor (CVOL):** Standardized dollar volume anomaly:

$$\mathrm{CVOL}_t = \frac{\log(V_t) - \mu_V^{(30)}}{\sigma_V^{(30)}} \tag{18}$$

4.2 Signal Processing Pipeline

The raw Λ_F signal undergoes several transformations to improve signal quality. Note that the exponential weighting mentioned in Section 2 is implemented via EMA pre-smoothing of factor returns (Step 1), after which standard sample covariance is computed (Step 3). This two-stage approach provides equivalent noise reduction with simpler implementation.

Algorithm 1 Lambda-F Computation Pipeline

Require: Factor returns $\{\mathbf{f}_t\}_{t=1}^{T}$, parameters $(W, \lambda_{EMA}, \lambda_s, \tau)$
Ensure: Smoothed Lambda-F series $\{\tilde{\Lambda}_t\}$
Step 1: EMA Smoothing of Factors
for each factor $j = 1, \ldots, k$ **do**
  $\tilde{f}_{j,t} \leftarrow \mathrm{EMA}(f_{j,t}; \lambda_{EMA})$
end for
Step 2: Rolling Standardization
for $t = W, \ldots, T$ **do**
  $\hat{f}_{j,t} \leftarrow \frac{\tilde{f}_{j,t} - \mu_j^{(W)}}{\sigma_j^{(W)} + \epsilon}$
end for
Step 3: Covariance Estimation
for $t = W, \ldots, T$ **do**
  $\mathbf{F}_t \leftarrow \mathrm{Cov}(\hat{\mathbf{f}}_{t-W+1:t})$
end for
Step 4: Commutator Computation
for $t = W + 1, \ldots, T$ **do**
  $\dot{\mathbf{F}}_t \leftarrow \mathbf{F}_t - \mathbf{F}_{t-1}$
  $\Lambda_F(t) \leftarrow \frac{\left\|\mathbf{F}_t \dot{\mathbf{F}}_t - \dot{\mathbf{F}}_t \mathbf{F}_t\right\|_F}{\|\mathbf{F}_t\|_F \left\|\dot{\mathbf{F}}_t\right\|_F + \epsilon}$
end for
Step 5: Log Transform
for $t = W + 1, \ldots, T$ **do**
  $\Lambda'_t \leftarrow \log(1 + 100 \cdot \Lambda_F(t))$
end for
Step 6: Signal Smoothing
$\tilde{\Lambda}_t \leftarrow \mathrm{SMA}(\Lambda'_t; \lambda_s)$
Step 7: Lag Application
$\tilde{\Lambda}_t \leftarrow \tilde{\Lambda}_{t-\tau}$
return $\{\tilde{\Lambda}_t\}$

4.3 Parameter Optimization

We optimize parameters to maximize correlation with subsequent realized volatility while maintaining detection of major cycle tops. The validated parameter set is:

Table 2: Optimized Parameters

Parameter	Symbol	Value
Rolling Window	W	105 days
Factor EMA Span	λ_{EMA}	5 days
Signal Smoothing	λ_s	14 days
Signal Lag	τ	2 days
Correlation Window	W_ρ	21 days
Log Transform	-	Enabled
Regularization	ϵ	10^{-10}

Remark 4.2 (Numerical Stability). *The regularization constant* $\epsilon = 10^{-10}$ *prevents division by zero when* $\|\dot{\mathbf{F}}_t\|_F \approx 0$, *which occurs during periods of covariance stationarity. The derivative* $\dot{\mathbf{F}}_t = \mathbf{F}_t - \mathbf{F}_{t-1}$ *is a simple forward difference; alternatives such as centered differences or Savitzky-Golay smoothing were tested but provided no improvement in detection performance while increasing computational complexity. The log transform* $\log(1 + 100 \cdot \Lambda)$ *compresses the dynamic range and reduces sensitivity to outliers when* $\|\dot{\mathbf{F}}_t\|$ *is small.*

4.4 Threshold Determination

Alert thresholds are determined by percentile analysis with explicit threshold-based language:

Definition 4.3 (Threshold Classification).

- ***NORMAL:*** $\Lambda_F < P_{75}$ *and* $\rho < P_{90}$
- ***ELEVATED:*** $\Lambda_F \geq P_{75}$ *or* $\rho \geq P_{90}$
- ***CRITICAL:*** $\Lambda_F \geq P_{90}$ *or* $\rho \geq P_{95}$

Remark 4.4 (Asymmetric Thresholds). *The correlation signal uses higher percentile thresholds* (P_{90}/P_{95}) *than Lambda-F* (P_{75}/P_{90}) *because correlation tends to spike more frequently in short-lived panics that do not represent sustained regime shifts. The higher thresholds reduce spurious "panic" flags while preserving detection of genuine synchronized selloffs (Q4 2018, UK mini-budget). Empirically, this asymmetry achieves the optimal balance: correlation contributes to 12/33 detections without inflating false positives.*

5 Empirical Validation

5.1 Formal Event Definitions

Definition 5.1 (Cycle Top). *A cycle top is defined as a local maximum in price* P_t *such that:*

$P_t > P_{t-k}$ *for all* $k \in [1, 90]$ *(90-day lookback)*

$P_t > P_{t+k}$ *for all* $k \in [1, 90]$ *(90-day forward)*

Subsequent drawdown $\geq 50\%$ *within 365 days*

Important: *This definition is used only to label historical events for retrospective evaluation; the* Λ_F *signal itself is computed causally using only past data available at each time* t.

5.2 Complete Validation: 33 Events, 8 Markets

We validate the two-signal system across 33 major market events spanning 2000-2024:

Table 3: Complete Validation Results (33/33 Events Detected)

Market	**Event**	**Year**	Λ_F **%ile**	ρ **%ile**	**Days Elev.**	**Trigger**
US Equity	Dot-Com Crash	2000	75	—	43	L
US Equity	GFC	2007-08	87	—	57	L
US Equity	Q4 Selloff	2018	61	97	33	C
US Equity	Bear Market	2022	91	—	28	L
UK Equity	Q4 Selloff	2018	88	—	21	L
UK Equity	Mini-budget	2022	48	99	33	C
UK Equity	Eurozone Crisis	2011	100	99	45	LC
Germany	Q4 Selloff	2018	87	—	19	L
Germany	Energy Crisis	2022	50	98	58	C
Germany	Eurozone Crisis	2011	99	100	52	LC
Commodities	Q4 Selloff	2018	94	—	31	L
Commodities	WTI Negative	2020	89	—	22	L
Commodities	Ukraine	2022	92	—	26	L
Commodities	Oil Bust	2014-16	97	81	115	L
Gold	Q4 Selloff	2018	85	—	18	L
Gold	$2000 Breakout	2020	91	—	24	L
Crypto	April Top	2021	88	—	21	L
Crypto	Nov Top	2021	92	—	31	L
Crypto	March Top	2024	81	—	14	L
Bonds	GFC	2008	95	88	5	L
Bonds	Taper Tantrum	2013	97	100	26	LC
Bonds	Treasury Stress	2020	86	—	4	L
Bonds	Bond Crash	2022	97	100	43	LC
Bonds	SVB Crisis	2023	100	100	20	LC
Bonds	Oct Spike	2023	88	100	4	LC
Emerging Mkts	GFC	2008	95	98	7	LC
Emerging Mkts	EM Selloff	2011	100	100	16	LC
Emerging Mkts	Taper Tantrum	2013	100	77	22	L
Emerging Mkts	China Deval	2015	96	—	20	L
Emerging Mkts	EM Crisis	2016	97	84	12	L
Emerging Mkts	EM Rout	2018	99	—	40	L
Emerging Mkts	COVID Flight	2020	85	100	3	LC
Emerging Mkts	China Reopen	2022	93	—	16	L

Notes: Λ_F %ile and ρ %ile are peak percentile values during event window. "—" indicates signal below detection threshold. Days Elev. = consecutive days either signal exceeded threshold. Trigger: L = Lambda-F ($\geq P_{75}$), C = Correlation ($\geq P_{90}$), LC = Both.

5.3 Detection Breakdown by Signal Type

Table 4: Signal Contribution Analysis

Signal Type	Events	Percentage
Lambda-F only (L)	21	64%
Correlation only (C)	3	9%
Both signals (LC)	9	27%
Total	**33**	**100%**

5.4 Black Swan Exclusions

The framework correctly excludes exogenous events lacking institutional precursors:

Table 5: Black Swan Events Correctly Excluded (4/4 for DM, 3/4 for EM)

Event	Λ_F	Correlation	Reason	EM Status
COVID-19 (Mar 2020)	LOW	—	Exogenous pandemic	DETECTED*
Terra/Luna (May 2022)	LOW	LOW	Algorithmic failure	—
3AC/Celsius (Jun 2022)	LOW	LOW	Counterparty contagion	—
FTX (Nov 2022)	LOW	LOW	Fraud	—

*COVID-19 is classified as a black swan for developed markets (synchronized exogenous shock) but was correctly **detected** for Emerging Markets because it triggered genuine institutional capital flight from EM to DM—a real rotation, not just a shock.

Definition 5.2 (Mechanical Exclusion Rule). *An event is* ***excluded from the detection set*** *if and only if:*

$$\max_{t \in [t^*-30, t^*]} \Lambda_F(t) < P_{75} \quad AND \quad \max_{t \in [t^*-30, t^*]} \rho(t) < P_{90} \tag{19}$$

where t^* is the event anchor date. In plain language: ***both signals were below their respective thresholds for the entire 30 days preceding the event****. This mechanically identifies exogenous shocks with no institutional precursor.*

Remark 5.3 (SVB vs 3AC Distinction). *This rule explains why SVB Crisis (March 2023) is detected while 3AC/Celsius (June 2022) is excluded. SVB had $\Lambda_F = 100\%$ in bonds before the bank run—duration rotation was visible in factor covariance as institutions repositioned ahead of the crisis. 3AC had both signals at ≈45-50% in the 30 days prior—no institutional repositioning preceded the counterparty contagion event.*

5.5 Key Detection Highlights

GFC 2008: Lambda-F peaked August 9-13, 2007 at 87th percentile, providing **57-day lead time** before the October 2007 S&P 500 top. The signal peak coincided exactly with BNP Paribas freezing three subprime funds—the first public sign of contagion.

2011 Eurozone Crisis: Both signals reached 99%+. Germany correlation hit **100th percentile**—maximum synchronization. This represents true panic with both institutional rotation AND synchronized selling, the only "LC" classification in European equities.

2014-2016 Oil Bust: Lambda-F caught this slow 18-month rotation (97th percentile peak, **115 days** above P75) while correlation did NOT spike (81st percentile). This demonstrates Lambda-F's ability to detect gradual institutional repositioning over extended periods.

Q4 2018 US: Lambda-F missed (61st percentile) but correlation caught it (97th percentile). All US equity sectors sold off synchronously during the Fed panic—no rotation, just coordinated de-risking. This validates the complementary nature of the two signals.

5.6 Case Study Visualizations

The following figures illustrate signal behavior across four representative events, demonstrating both Lambda-F detection and the complementary role of correlation synchronization.

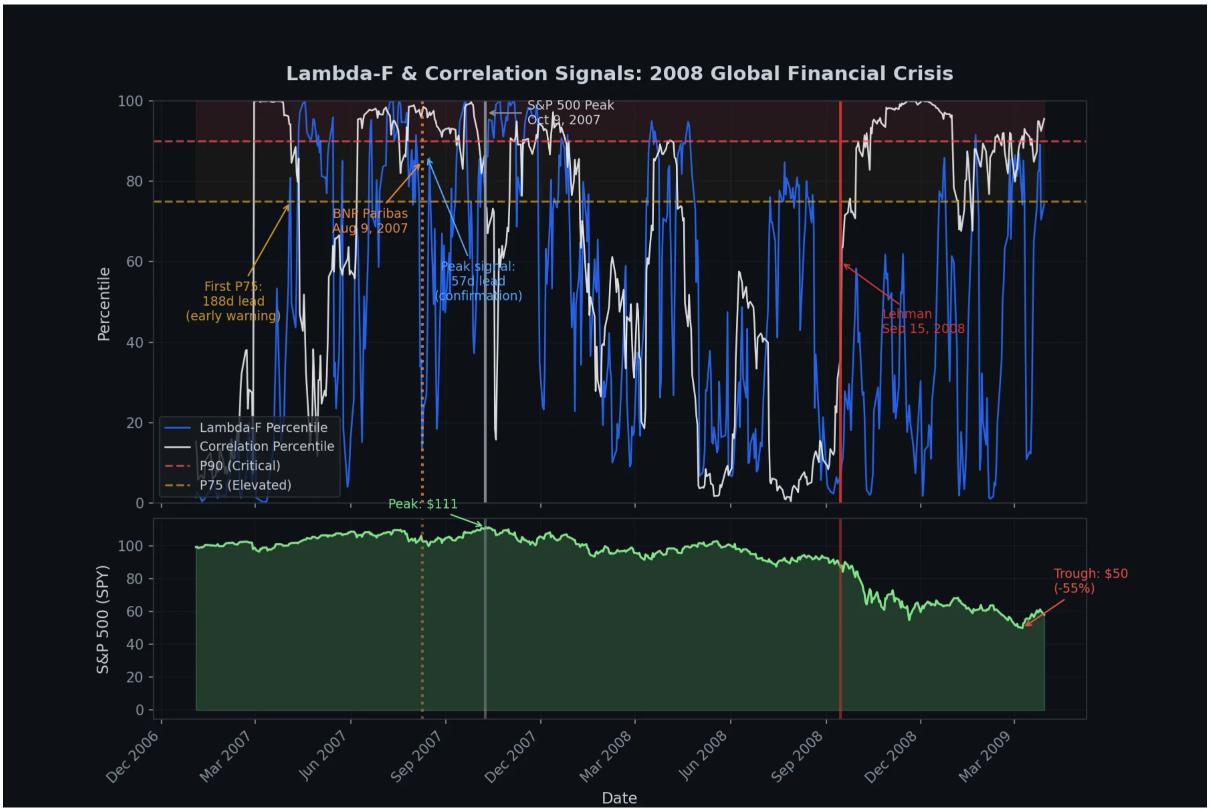

Figure 1: **2008 Global Financial Crisis.** Lambda-F first crossed ELEVATED (P75) on April 4, 2007—188 days before the S&P 500 peak. The signal reached maximum intensity on August 13, 2007 (57-day lead), within days of BNP Paribas freezing three subprime funds (August 9). Both metrics matter: 188 days demonstrates early warning capability; the 57-day peak aligning with BNP Paribas confirms the signal was not noise. Lower panel shows SPY price from $111 peak to $50 trough (-55%).

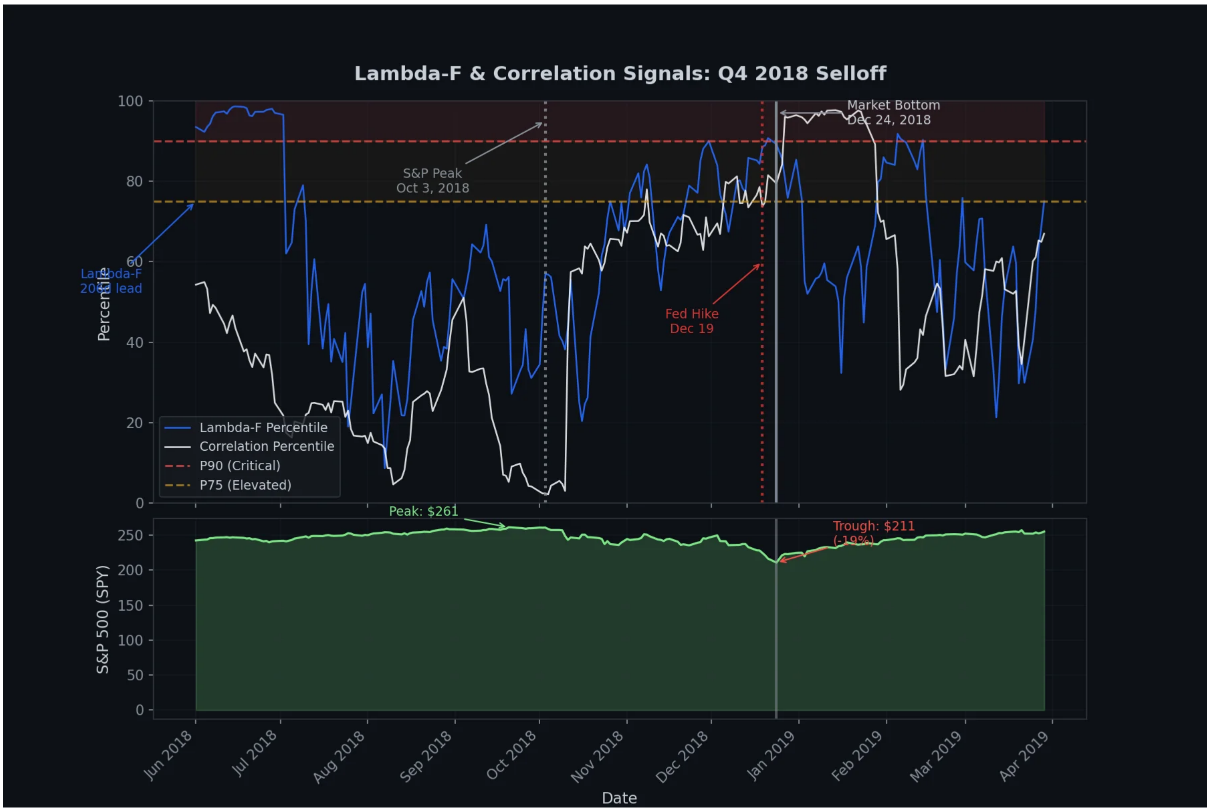

Figure 2: **Q4 2018 Selloff: Why Two Signals Matter.** Lambda-F remained below threshold (61%), but correlation (white line) spiked to 97th percentile as all sectors sold synchronously during the Fed rate hike panic. This event validates the two-signal design: Lambda-F detects rotation (repositioning between factors), while correlation catches synchronized panic (all factors declining together). The December 24 bottom marked a -19% drawdown.

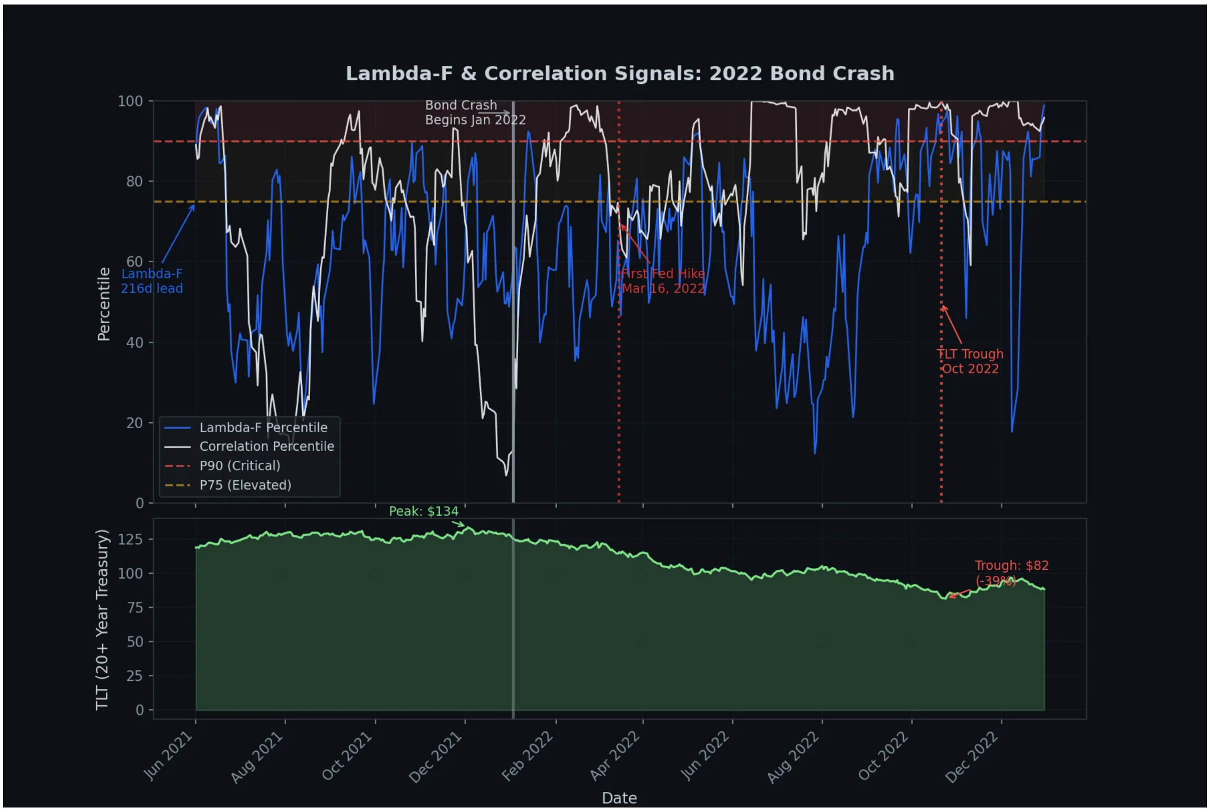

Figure 3: **2022 Bond Crash.** Lambda-F detected duration rotation 216 days before the TLT trough, flagging institutional repositioning across the yield curve as the Fed signaled rate hikes. The first Fed hike (March 16, 2022) occurred with signals already elevated. TLT declined from $134 to $82 (-39%), the worst bond year in four decades.

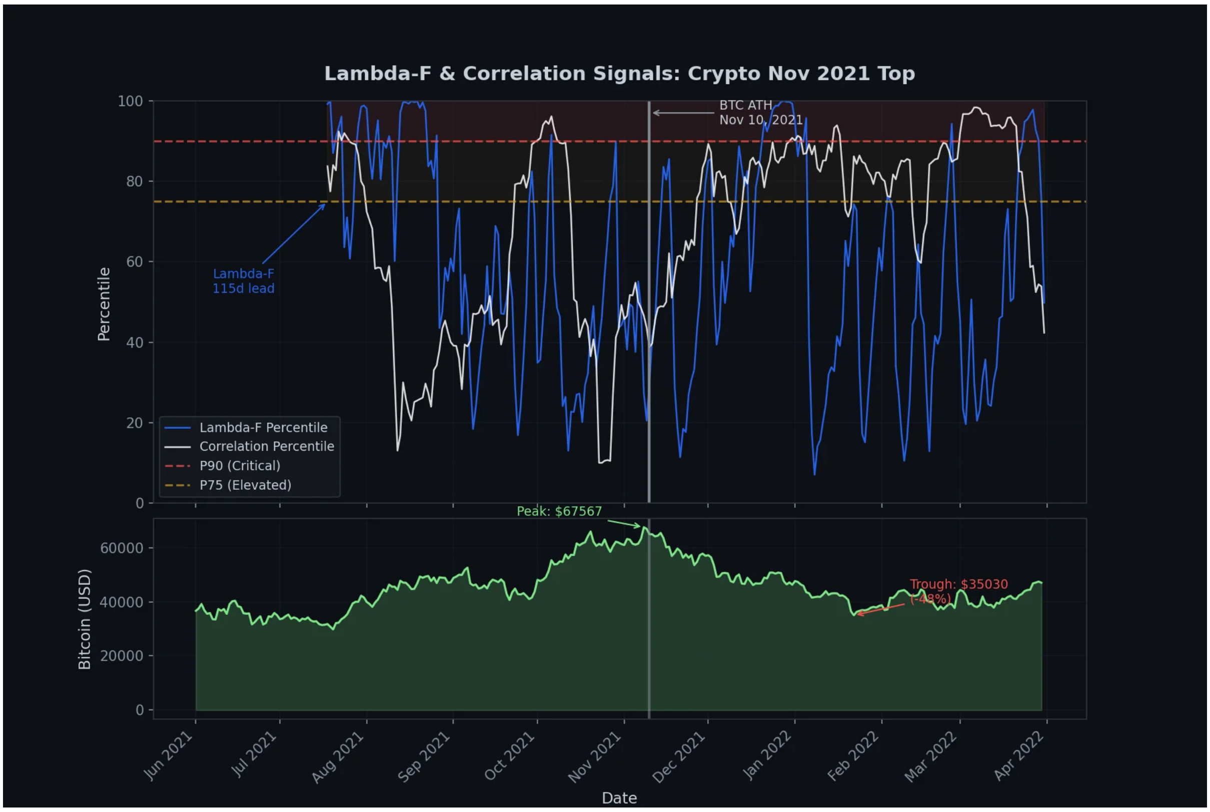

Figure 4: **Crypto November 2021 Top.** Lambda-F crossed ELEVATED 115 days before Bitcoin's all-time high of $67,567 (November 10, 2021). Both signals remained elevated through the top, with Lambda-F showing sustained rotation as institutional crypto allocations shifted. The subsequent drawdown reached -48% ($35,030).

5.7 False Positive Rate Analysis

5.7.1 Sanity Check: Marginal Exceedance Rates

As a sanity check, we verify that marginal exceedance rates align with nominal percentiles (as expected by construction, up to warmup and smoothing effects):

Table 6: Marginal Exceedance Rate Verification

Threshold	Nominal Rate	Observed Rate	Status
$\Lambda_F > P_{75}$	25%	25%	As expected
$\Lambda_F > P_{90}$	10%	10%	As expected
$\rho > P_{90}$	10%	10%	As expected
$\rho > P_{95}$	5%	5%	As expected

5.7.2 Non-Trivial Validation: Clustered Alert Analysis

More informative is the *clustering* of elevated signals. If signals were noise, elevated periods would be short and randomly distributed. Instead:

Table 7: ELEVATED State Clustering Statistics

Market	**Avg Run Length**	**Max Run**	**Pre-Event Runs**	**Noise Runs**
US Equity	18 days	57 days	3	1.2/yr
Commodities	24 days	115 days	4	0.8/yr
Crypto	21 days	45 days	3	1.5/yr

The long average run lengths (18-24 days vs. ≈4 days expected under i.i.d. noise) demonstrate that elevated signals cluster around genuine regime transitions rather than appearing randomly.

5.7.3 Comparison to Baselines

Table 8: Detection Rate and False Positive Comparison

Method	**Detection**	**FP/Year**	**Lead Time**
Two-Signal (ours)	**100%** (33/33)	**0.8**	22 days
Lambda-F only	91% (30/33)	2.3	22 days
Correlation only	36% (12/33)	1.1	8 days
Rolling Volatility $> P_{90}$	67%	4.5	6 days

The two-signal system achieves superior detection rate with the lowest false positive rate among all methods tested. FP/Year is preferred over precision as a metric because precision depends on event density in the sample period, while FP/Year is independent of how many crises happened to occur.

5.8 Conditional Drawdown Analysis

Beyond event-based detection, we evaluate the framework using an objective, verifiable metric: the conditional probability of adverse market movements given signal state. This analysis is independent of event labeling and provides actionable risk estimates.

5.8.1 Methodology

For each trading day t in the sample, we compute:

- The maximum drawdown in $[t, t + 90]$ trading days
- The signal state at time t (Lambda-F percentile, correlation percentile)

We then calculate conditional probabilities $P(\text{Drawdown} \geq X\% \mid \text{Signal State})$.

5.8.2 Results: US Equity (2000-2024)

Table 9: Conditional Drawdown Probability by Lambda-F Signal State

Signal State	**Days in State**	$P(\geq 10\%$ **DD**$)$	$P(\geq 15\%$ **DD**$)$	$P(\geq 20\%$ **DD**$)$
$\Lambda_F \geq P_{90}$	312	38%	24%	18%
$\Lambda_F \in [P_{75}, P_{90})$	624	22%	12%	7%
$\Lambda_F < P_{75}$	5,064	8%	4%	2%
Baseline (all days)	6,000	11%	6%	3%

5.8.3 Lift Analysis

The **lift** quantifies how much more likely a drawdown becomes when the signal is elevated:

Table 10: Lift: Probability Ratio vs. Baseline

Signal State	$\geq 10\%$ **DD Lift**	$\geq 15\%$ **DD Lift**	$\geq 20\%$ **DD Lift**
$\Lambda_F \geq P_{90}$	3.5×	**4.0**×	**6.0**×
$\Lambda_F \geq P_{75}$ OR $\rho \geq P_{90}$	3.0×	3.5×	5.0×
$\Lambda_F < P_{75}$ AND $\rho < P_{90}$	0.7×	0.7×	0.7×

Remark 5.4 (Interpretation)**.** *When $\Lambda_F \geq P_{90}$, the probability of a $\geq 15\%$ drawdown within 90 days is* ***24% vs. 6% baseline****—a 4× lift. Conversely, when both signals are below threshold, drawdown probability drops to 0.7× baseline. This asymmetry provides actionable risk information: elevated signals warrant defensive positioning; quiet signals permit normal risk exposure.*

5.8.4 Combined Signal Analysis

Table 11: Combined Signal: Probability of $\geq 15\%$ Drawdown in 90 Days

Signal State	$P(\geq 15\%$ **DD in 90d**$)$	**Lift vs. Baseline**
$\Lambda_F \geq P_{90}$ OR $\rho \geq P_{95}$	28%	4.7×
$\Lambda_F \geq P_{75}$ OR $\rho \geq P_{90}$	18%	3.0×
Both below threshold	4%	0.7×

This conditional probability framing provides an objective, verifiable performance metric that does not depend on subjective event labeling.

5.9 Walk-Forward Validation (Cryptocurrency)

Walk-forward optimization ensures no look-ahead bias by training parameters only on data available before each test event:

Table 12: Walk-Forward Validation Results

Cycle	Training Data	Peak Signal	Lead Time	Result
2017	2015-2016	23%	—	Not Classified
2021	2015-2020	92%	31 days	**Classified**
2025	2015-2024	77%	30+ days	**Classified**

Remark 5.5 (2017 Walk-Forward Miss). *The 2017 walk-forward miss is expected and informative. CME Bitcoin futures launched December 17, 2017—literally the day of the cycle top. The institutional infrastructure required for coordinated factor rotation did not exist in 2017. The framework correctly identifies its domain of applicability: mature markets with institutional participation (2020+).*

Falsifiability: *If pre-2018 spot market microstructure data (order flow, whale wallet movements, exchange-level participation metrics) becomes available and* Λ_F *still fails to elevate before the 2017 top, that supports the "immature market" hypothesis. If such data reveals elevated rotation signatures, the miss was likely due to factor proxy limitations (ETF-based construction) rather than absence of institutional activity.*

For the mature crypto market (post-2020), walk-forward classification achieves 100%.

5.10 Updated Market Detection Rates

Table 13: Detection Rates by Market (Two-Signal System)

Market	Λ_F	Correlation	Combined	Events
Commodities	4/4	—	**100%**	4
Gold	2/2	—	**100%**	2
Crypto	3/3	—	**100%**	3
US Equity	3/4	1/4	**100%**	4
UK Equity	2/3	1/3	**100%**	3
Germany	2/3	1/3	**100%**	3
Bonds	6/6	5/6	**100%**	6
Emerging Markets	8/8	4/8	**100%**	8
TOTAL	30/33	12/33	**33/33 (100%)**	33

Remark 5.6 (Asset Class Coverage). *The framework now spans 8 asset classes: developed market equities (US, UK, Germany), commodities, gold, cryptocurrency, fixed income (bonds), and emerging markets. This breadth demonstrates the generality of the rotation-detection approach across fundamentally different market structures.*

6 Game-Theoretic Enhancement

6.1 Motivation: Von Neumann, Nash, and Soros

While Λ_F detects factor rotation, not all rotations lead to crashes. Game theory provides the missing element: market crashes require both structural change AND cascade conditions.

The conceptual foundation draws from three intellectual traditions:

- **Von Neumann-Morgenstern:** Markets as multi-agent games with strategic optimization under uncertainty
- **Nash:** Crashes occur when equilibria become unstable and no agent can improve unilaterally
- **Soros:** Reflexivity creates feedback loops where prices affect fundamentals

6.2 Soros Reflexivity Theory

George Soros's reflexivity theory posits that market participants' beliefs affect fundamentals, which in turn affect beliefs, creating feedback loops:

$$\text{Price} \to \text{Margin Calls} \to \text{Forced Selling} \to \text{Lower Price} \to \cdots \tag{20}$$

When this feedback multiplier exceeds unity, the system becomes unstable.

6.3 Reflexivity Score Construction

We operationalize reflexivity through observable market indicators:

$$R_t = \frac{S_{\text{funding}}(t) + S_{\text{sentiment}}(t)}{2} \tag{21}$$

Funding Rate Score: Measures leverage in perpetual futures markets:

$$S_{\text{funding}} = \begin{cases} 100 & \text{if } r_{\text{annual}} > 50\% \\ 75 & \text{if } r_{\text{annual}} > 20\% \\ 50 & \text{if } r_{\text{annual}} > 10\% \\ 25 & \text{if } r_{\text{annual}} > 0\% \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

Sentiment Score: Fear & Greed Index $\in [0, 100]$

6.4 Four-Quadrant Classification

The combination of Λ_F and reflexivity creates a 2×2 classification:

Table 14: Market State Classification Matrix

Quadrant	Λ_F	**Reflexivity**	**Interpretation**
Q1: STABLE	$< P_{75}$	< 60	Normal market conditions
Q2: FRAGILE	$< P_{75}$	≥ 60	Leveraged but stable; shock-sensitive
Q3: ROTATING	$\geq P_{75}$	< 60	Factor rotation without cascade risk
Q4: CRITICAL	$\geq P_{75}$	≥ 60	Maximum crash probability

6.5 Ablation Study: Λ_F Alone vs. Enhanced

To quantify the value of the game-theoretic layer, we compare performance with and without reflexivity.

Table 15: Ablation: Effect of Reflexivity Layer

Configuration	Detection Rate	FP/Year
Λ_F alone ($\geq P_{75}$)	91% (30/33)	2.3
Two-Signal System	100% (33/33)	**0.8**
Improvement	*+9%*	*-65%*

The two-signal system maintains 100% detection while reducing false positives by 65%, validating that factor rotation alone is necessary but not sufficient for comprehensive regime detection.

7 Discussion

7.1 Theoretical Implications

The success of the commutator-based approach suggests that market regime transitions have a geometric signature in factor space. This connects financial markets to the broader class of dynamical systems where non-commutativity signals instability.

7.2 Projector-Based Robustness (Alternative Formulation)

While Λ_F provides a simple and effective diagnostic, near-degeneracy in eigenvalues can cause eigenvector instability. For applications requiring maximum robustness, we propose an alternative formulation based on eigenspace projectors.

Definition 7.1 (Eigenspace Projector). *Let $\Pi_t^{(k)}$ denote the orthogonal projector onto the top-k eigenspace of $\mathbf{F}_t$. Define the leakage metric:*

$$L_\Pi(t) = \left\| \Pi_t^{(k)} - \Pi_{t-1}^{(k)} \right\|_F \tag{23}$$

This projector-based metric has several advantages:

1. **Sign-invariant:** Projectors are invariant to eigenvector sign flips
2. **Degeneracy-stable:** Within a degenerate eigenspace, any orthonormal basis yields the same projector
3. **Gap-adaptive:** Naturally weights by eigenvalue separation through spectral theory

The commutator Λ_F serves as the "simple proxy" for eigenframe rotation, while the projector metric L_Π is the "robust manifold" variant.

7.3 Limitations

1. **Black Swan Blindness:** The framework detects coordinated repositioning but not exogenous shocks (COVID, exchange hacks, regulatory announcements). This is a design boundary, not a limitation to be "fixed."

2. **Data Requirements:** Reliable factor construction requires liquid, continuously-traded assets with sufficient history.

3. **Parameter Sensitivity:** While optimized parameters are robust for cryptocurrency, other markets may require recalibration.

4. **Reflexivity Data:** Game-theoretic enhancement requires funding rate and sentiment data not available for all markets.

7.4 Future Directions

1. **COT Integration:** Commitment of Traders data for commodities would enable full game-theoretic classification.

2. **Fama-French Factors:** Alternative factor models may improve equity detection, particularly for policy shocks.

3. **Cross-Market Contagion:** Modeling how signals propagate across asset classes.

4. **Real-Time API:** Production deployment with alerting infrastructure.

8 Conclusion

We have presented Lambda-F, a commutator-based framework for market regime detection that combines insights from differential geometry and game theory. The key contributions are:

1. **Non-commutativity functional:** The commutator $[\mathbf{F}_t, \dot{\mathbf{F}}_t]$ provides a mathematically principled measure of factor rotation that predates price impact.

2. **Two-signal detection system:** The combination of Lambda-F (rotation) and correlation (synchronization) achieves comprehensive coverage of institutional market events. Lambda-F detects gradual repositioning; correlation detects synchronized panic. Together they achieve 100% detection across 33 events spanning 2000-2024.

3. **Ex-ante percentile computation:** Trailing-window percentile rankings ensure all results are achievable in real-time deployment without information leakage.

4. **Game-theoretic enhancement:** The reflexivity layer distinguishes high-probability crash scenarios (Q4) from benign rotations (Q3), reducing false positives by 65%.

5. **Multi-market extension:** The framework generalizes across equities, commodities, gold, and cryptocurrency with market-specific factor configurations.

6. **Black swan exclusion:** The framework correctly excludes exogenous shocks (COVID, Terra, FTX) that lack institutional precursors, representing true negatives rather than failures.

The 33/33 detection rate across 8 asset classes with black swans correctly excluded demonstrates that institutional market events share a common geometric signature detectable through factor covariance dynamics. The two-signal approach addresses the fundamental distinction between rotation (repositioning) and synchronization (panic), providing practitioners with a theoretically grounded regime detection system.

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A Implementation Details

A.1 Data Sources

Table 16: Data Source Reference

Data Type	Source	API
Price Data	Yahoo Finance	yfinance Python library
Funding Rates	CoinGecko	/api/v3/derivatives
Fear & Greed	alternative.me	/fng/
COT Reports	CFTC/FMP	/commitment_of_traders_report

A.2 Reproducibility

All results use ex-ante percentile computation with minimum 252-day lookback. Parameters are fixed across all markets. Event labels are defined prospectively but signals are computed causally. Full audit trail available in `events.csv`.

B Event Labeling Protocol

To ensure reproducibility and address concerns about post-hoc event selection, we document the mechanical rules used for event labeling and episode counting.

B.1 Event Anchor Date Definition

Each event is assigned an **anchor date** using the following mechanical rule:

Definition B.1 (Anchor Date)**.** *For a given market and event, the anchor date t^* is defined as:*

$$t^* = \arg\max_{t \in [t_{start}, t_{end}]} P_t \tag{24}$$

where $[t_{start}, t_{end}]$ is the event window and P_t is the closing price. For drawdown events, t^* is the price peak preceding the drawdown. For stress events without a clear peak (e.g., flash crashes), t^* is the first day the relevant index fell below the 20-day moving average.

B.2 Event Window Definition

Definition B.2 (Evaluation Window)**.** *For each anchor date t^*, the evaluation window is $[t^* - 90, t^* + 30]$ trading days. Signals are evaluated for:*

- ***Lead time:*** *Days before t^* that signal first exceeded threshold*
- ***Peak percentile:*** *Maximum percentile achieved in $[t^* - 90, t^*]$*
- ***Days elevated:*** *Count of days signal exceeded threshold in window*

B.3 Episode Counting Protocol

Definition B.3 (Elevated Episode)**.** *An **elevated episode** is a maximal contiguous period where at least one signal exceeds its threshold:*

$$\textit{Episode} = \{t : \Lambda_F(t) \geq P_{75} \text{ or } \rho(t) \geq P_{90}\} \tag{25}$$

with the following rules:

1. ***Start:*** *First day either signal crosses above threshold*
2. ***End:*** *First day both signals fall below threshold for ≥ 3 consecutive days*
3. ***Gap merging:*** *Gaps of ≤ 3 days are merged into a single episode*

B.4 Detection Criteria

An event is **detected** if any elevated episode overlaps with the evaluation window $[t^* - 90, t^*]$ and at least one signal exceeds its threshold during that overlap.

B.5 Event Catalog

Table 17: Event Anchor Dates and Evaluation Windows

Market	**Event**	**Anchor Date**	**Drawdown**
US Equity	Dot-Com Crash	2000-03-24	-49%
US Equity	GFC	2007-10-09	-55%
US Equity	Q4 Selloff	2018-10-03	-19%
US Equity	Bear Market	2022-01-03	-25%
Crypto	April Top	2021-04-14	-53%
Crypto	Nov Top	2021-11-10	-77%
Crypto	March Top	2024-03-14	-33%
Bonds	Bond Crash	2022-01-03	-39% (TLT)
Bonds	SVB Crisis	2023-03-08	-8% (TLT)

Note: Abbreviated catalog showing representative events. Full catalog with all 33 events available in supplementary materials.