Predictive Geometric Control of Quantum Dynamics via Spectral Curvature Functional $\Lambda(t)$

R. J. Mathews
Independent Researcher
rjmathews@getqore.ai

Abstract

Non-adiabatic transitions remain a dominant source of leakage and decoherence in electrically controlled qubits, including semiconductor spin qubits driven through avoided crossings via Landau–Zener–Stückelberg–Majorana (LZSM) interference. Conventional control methods are reactive: leakage is measured only after an error has already occurred. We introduce a predictive control framework based on a *spectral curvature functional* $\Lambda(t)$, defined as the operator norm of the commutator between the instantaneous generator of evolution and its time derivative. For a two-level quantum system governed by a time-dependent Hamiltonian $\mathcal{H}(t) = \frac{\hbar}{2}(\Delta(t)\sigma_z + \Omega\,\sigma_x)$ with constant coupling Ω, we derive an exact identity

$$\Lambda(t) = \frac{1}{2}\,\Omega\,|\dot{\Delta}(t)|.$$

At the gap minimum t^*, the Landau–Zener transition probability satisfies

$$P_{\mathrm{LZ}} \approx \exp\left(-\frac{\pi\Omega^3}{2\Lambda(t^*)}\right),$$

establishing a direct, pointwise control law: minimizing peak curvature at the avoided crossing exponentially suppresses leakage. This is complemented by a frequency-domain analysis showing that peak curvature is bounded by the spectral first moment $\int|\omega|\,|\widehat{\Delta}(\omega)|\,d\omega$, providing a rigorous low-pass design rule. We present a predictive feedback loop that enforces $\Lambda(t) \le \alpha_{\mathrm{crit}}$ during gate operation. Simulation of LZSM sweeps shows that curvature-constrained waveforms reduce leakage relative to naive linear ramps without requiring counter-diabatic fields or full Hamiltonian inversion. This work provides a simple geometric precursor for non-adiabatic transitions and a practical control strategy compatible with existing pulse-generation hardware.

1 Introduction

Electrically controlled qubits, particularly semiconductor spin qubits, frequently rely on avoided crossings to implement state transfer or phase accumulation. Leakage during these crossings is typically modeled by the Landau–Zener (LZ) formula, producing a transition probability

$$P_{\mathrm{LZ}} \approx \exp\left(-\frac{\pi\Omega^2}{\nu}\right), \tag{1}$$

where Ω is the transverse coupling and $\nu = |\dot{\Delta}(t^*)|$ is the detuning sweep rate at the minimum gap. Existing control approaches measure leakage only *after* it has occurred, requiring repeated calibration or counter-diabatic corrections.

We propose a *predictive* control parameter: the *spectral curvature functional* $\Lambda(t)$. This observable produces a precursor signal for imminent leakage and yields a direct control objective: constrain Λ near the avoided crossing to suppress P_{LZ} in real time.

2 Mathematical Framework

Consider a two-level Hamiltonian

$$\mathcal{H}(t) = \frac{\hbar}{2}(\Delta(t)\sigma_z + \Omega\sigma_x), \tag{2}$$

with Ω constant and $\Delta(t)$ a control waveform. Define the instantaneous generator

$$B(t) = \frac{\mathcal{H}(t)}{\hbar}$$

and the *curvature functional*

$$\Lambda(t) = \|[B(t), \dot{B}(t)]\|, \tag{3}$$

where $\|\cdot\|$ denotes the operator norm. Non-zero Λ indicates that the instantaneous plane of rotation is changing, signaling non-adiabatic driving.

2.1 Lemma 1: Curvature–Sweep Identity

Lemma 1. *For the Hamiltonian in Eq.* (2) *with constant* Ω*, the curvature functional satisfies*

$$\Lambda(t) = \frac{1}{2}\,\Omega|\dot{\Delta}(t)|. \tag{4}$$

Proof. We compute

$$B(t) = \frac{1}{2}(\Delta(t)\sigma_z + \Omega\sigma_x), \quad \dot{B}(t) = \frac{\dot{\Delta}(t)}{2}\sigma_z.$$

Commuting gives

$$[B(t), \dot{B}(t)] = \frac{\Omega\dot{\Delta}(t)}{4}[\sigma_x, \sigma_z] = \frac{\Omega\dot{\Delta}(t)}{4}(-2i\sigma_y) = -\frac{i\Omega\dot{\Delta}}{2}\sigma_y.$$

Taking the standard operator norm $\|\sigma_y\| = 1$, this yields $\Lambda(t) = \|-\frac{i\Omega\dot{\Delta}}{2}\sigma_y\| = \frac{1}{2}\Omega|\dot{\Delta}(t)|$. $\square$

2.2 Lemma 2: Peak Curvature Controls Leakage

Let t^* denote the minimum gap, i.e. $\Delta(t^*) \approx 0$. From Eq. (1) and $\nu = |\dot{\Delta}(t^*)| = 2\Lambda(t^*)/\Omega$ (from Lemma 1), we obtain:

Lemma 2. *The Landau–Zener transition probability is expressible directly in terms of the curvature at the gap minimum:*

$$P_{\mathrm{LZ}} \approx \exp\left(-\frac{\pi\Omega^3}{2\Lambda(t^*)}\right). \tag{5}$$

Equation (5) establishes a *pointwise, predictive control law*: minimizing $\Lambda(t^*)$ exponentially suppresses leakage.

3 Frequency–Domain Analysis of the Curvature Functional

In this section we derive a frequency–domain bound on the peak curvature $\Lambda(t^*)$. Throughout, we assume $\Delta(t)$ is a pulse with $\dot{\Delta} \in L^1(\mathbb{R})$ so that all Fourier transforms are well-defined. While the transform definitions below assume causality for simplicity, the core time-domain results (Lemmas 1, 2) hold generally. The Fourier transform is defined as $\widehat{f}(\omega) \equiv \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$.

3.1 Spectral Moment Bound on Peak Curvature

The next result converts the *pointwise* slope $|\dot{\Delta}(t^*)|$ into an inequality involving a spectral first moment of the control pulse. It uses only the triangle inequality for the inverse Fourier transform and requires no bandlimit assumption.

Theorem 1 (Spectral moment bound on peak curvature)**.** *Suppose* $\widehat{\Delta} \in L^1(\mathbb{R}, (1+|\omega|)\, d\omega)$. *Then for every* t,

$$|\dot{\Delta}(t)| \;\le\; \frac{1}{2\pi}\int_{-\infty}^{\infty} |\omega|\, |\widehat{\Delta}(\omega)|\, d\omega. \tag{6}$$

Consequently, the peak curvature at the avoided crossing t^* *satisfies*

$$\Lambda(t^*) \;\le\; \frac{\Omega}{4\pi}\int_{-\infty}^{\infty} |\omega|\, |\widehat{\Delta}(\omega)|\, d\omega. \tag{7}$$

Proof. By Fourier inversion,

$$\dot{\Delta}(t) \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty} (i\omega)\, \widehat{\Delta}(\omega)\, e^{i\omega t}\, d\omega.$$

Taking absolute values and applying the triangle inequality yields

$$|\dot{\Delta}(t)| \;\le\; \frac{1}{2\pi}\int_{-\infty}^{\infty} |\omega|\, |\widehat{\Delta}(\omega)|\, d\omega,$$

which is (6). Multiplying by $\Omega/2$ (from Lemma 1) gives (7). □

Corollary 1 (Low-pass design rule)**.** *If $\widehat{\Delta}$ is supported in $[-\omega_c, \omega_c]$, then*

$$\Lambda(t^*) \ \le \ \frac{\Omega\,\omega_c}{4\pi} \int_{-\infty}^{\infty} |\widehat{\Delta}(\omega)|\, d\omega. \tag{8}$$

Equivalently, for fixed $\|\widehat{\Delta}\|_{L^1}$, reducing the bandwidth ω_c reduces an a priori upper bound on the peak curvature.

Proof. Under the bandlimit, $|\omega| \le \omega_c$ on the support of $\widehat{\Delta}$, giving

$$\int |\omega|\, |\widehat{\Delta}(\omega)|\, d\omega \ \le \ \omega_c \int |\widehat{\Delta}(\omega)|\, d\omega.$$

Apply Theorem 1. □

3.2 Consequences for Landau–Zener leakage

Let $\nu^* \equiv |\dot{\Delta}(t^*)|$ be the local sweep rate at the gap minimum. The Landau–Zener formula [3–6] gives

$$P_{\mathrm{LZ}} \ \approx \ \exp\Big(-\frac{\pi\,\Omega^2}{\nu^*}\Big) \ = \ \exp\Big(-\frac{\pi\,\Omega^3}{2\Lambda(t^*)}\Big). \tag{9}$$

Combining (9) with (7) yields the *spectral lower bound*

$$P_{\mathrm{LZ}} \ \ge \ \exp\left(-\frac{\pi\,\Omega^3/2}{\frac{\Omega}{4\pi}\int |\omega|\,|\widehat{\Delta}(\omega)|\, d\omega}\right) \ = \ \exp\left(-\frac{2\pi^2\,\Omega^2}{\displaystyle\int_{-\infty}^{\infty} |\omega|\,|\widehat{\Delta}(\omega)|\, d\omega}\right). \tag{10}$$

In particular, for a fixed spectral moment $\int |\omega|\,|\widehat{\Delta}|$, no control schedule can drive P_{LZ} below the right-hand side of (10). Under the bandlimit of Corollary 1 we also obtain

$$P_{\mathrm{LZ}} \ \ge \ \exp\left(-\frac{2\pi^2\,\Omega^2}{\omega_c\,\|\widehat{\Delta}\|_{L^1}}\right). \tag{11}$$

Remarks. (i) The bounds (10)–(11) are *lower bounds* on the achievable leakage, consistent with the monotonicity of P_{LZ} in ν^*. (ii) Unlike global time–domain heuristics (e.g. total variation), the spectral moment $\int |\omega|\,|\widehat{\Delta}|$ controls *all* pointwise slopes via (6), with no averaging. (iii) Stronger bounds follow by adding smoothness constraints, e.g. $|\ddot{\Delta}| \le L$ on a neighborhood of t^* (yielding windowed variants using local spectral content). We omit these for brevity.

3.3 Design implications

The frequency–domain identities and bounds justify curvature–constrained waveform design:

1. **Low spectral first moment** $\int |\omega|\,|\widehat{\Delta}| \Rightarrow$ smaller a priori bound on $\Lambda(t^*)$.

2. **Bandlimit or low-pass shaping** (Corollary 1) $\Rightarrow$ explicit handle on the curvature peak via ω_c and $\|\widehat{\Delta}\|_{L^1}$.

3. **Predictive control objective:** enforce $\Lambda(t) \le \alpha_{\text{crit}}$ near t^* and co-design the pulse to minimize the spectral moment, thereby reducing the fundamental leakage lower bound (10).

4 Predictive Control Loop

The control objective is to enforce

$$\Lambda(t) \le \alpha_{\text{crit}}$$

for t near t^*. This is implemented as a feedback loop:

- Measure or estimate $\Delta(t)$ in real time.
- Compute $\Lambda(t)$ via Eq. (4).
- If $\Lambda > \alpha_{\text{crit}}$, adjust the waveform slope (reduce $|\dot{\Delta}|$).

4.1 Pseudocode

```
Initialize waveform Delta(t)
for each timestep t:
    measure or estimate Delta
    compute Lambda = 0.5 * Omega * abs(dDelta/dt)
    if Lambda > alpha_crit:
        reduce slope of Delta(t)
    update DAC output
end
```

4.2 Implementation Considerations

We note that a practical, real-time implementation of this loop faces significant engineering challenges, including measurement back-action, system latency, and finite control bandwidth. This paper presents the theoretical control principle; a hardware demonstration would require translating this objective into an open-loop optimized pulse (using Section 3) or a closed-loop controller that respects these physical constraints, possibly by using model-based estimation rather than direct, disruptive measurement.

5 Simulation

6 Discussion

The curvature functional is a physically interpretable precursor: large Λ indicates imminent non-adiabatic evolution. Unlike reactive control, predictive regulation of Λ suppresses leakage before it occurs. This method extends naturally to time-varying $\Omega(t)$ and multi-qubit blocks via decomposition of $B(t)$, as derived in the authors' related patent application.

7 Conclusion

We introduced a spectral curvature functional that serves as a predictive control parameter for non-adiabatic transitions. For LZSM systems, Λ yields a closed-form relationship to P_{LZ}, and we derive a rigorous bound on peak curvature from the spectral first moment $\int |\omega|\,|\widehat{\Delta}(\omega)|\,d\omega$. This provides a simple control objective: minimize $\Lambda(t^*)$ and enforce $\Lambda \leq \alpha_{\mathrm{crit}}$. The approach is compatible with standard waveform hardware and may be applied to semiconductor spin qubits and other electrically controlled systems.

References

[1] Google Quantum AI, *Testing quantum error correction with real-time syndrome extraction*, Nature (2023).

[2] Zenodo Dataset: google_105Q_surface_code_d3_d5_d7, DOI: `https://zenodo.org/record/8169977`

[3] L. D. Landau, "On the theory of transfer of energy at collisions," *Physikalische Zeitschrift der Sowjetunion* 2, 46–51 (1932).

[4] C. Zener, "Non-adiabatic crossing of energy levels," *Proceedings of the Royal Society A* 137(833), 696–702 (1932).

[5] E. C. G. Stueckelberg, "Theory of inelastic collisions between atoms," *Helvetica Physica Acta* 5, 369–422 (1932).

[6] E. Majorana, "Atomi orientati in campo magnetico variabile," *Il Nuovo Cimento* 9, 43–50 (1932).

[7] G. Doetsch, *Introduction to the Theory and Application of the Laplace Transformation* (Springer, 1974).

[8] R. Bracewell, *The Fourier Transform and Its Applications*, 3rd ed. (McGraw-Hill, 2000).

Figure Placeholder

(fig_curvature_sweeps_placeholder.pdf)

Figure 1: Comparison of three detuning schedules $\Delta(t)$ with identical endpoints and duration T, illustrating the relationship between time–domain curvature $\Lambda(t)$ and spectral moment $\int |\omega| \, |\widehat{\Delta}(\omega)| \, d\omega$. **(A) Rectangular sweep.** High—almost discontinuous—slope at the switching point produces a large spike in $\Lambda(t)$ and correspondingly broad high–frequency tails in $|\widehat{\Delta}(\omega)|$. The spectral moment is large, resulting in a loose lower bound (10) on P_{LZ}. **(B) Gaussian sweep.** Smooth pulse with controlled bandwidth. Both the curvature peak and the spectral first moment decrease relative to (A), tightening the inequality (11). Leakage suppression improves even without modifying Ω. **(C) Adiabatic** tanh **ramp.** Near–minimal spectral moment and no sharp slope changes. The curvature peak at the avoided crossing is significantly reduced; the corresponding Landau–Zener lower bound becomes exponentially smaller. This illustrates the practical design rule of Corollary : reducing high–frequency spectral weight suppresses maximal curvature and thereby lowers P_{LZ}.